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Novel high-precision explicit equations for pipe diameter in turbulent flow via modified rough model method (MRMM)

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A R T I C L E I N F O

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A B S T R A C T

This study addresses key hydraulic engineering challenges in turbulent pipe flow - computing flow rate (Q), hydraulic energy slope S_f, and pipe diameter (D) - by introducing the Modified Rough Model Method (MRMM). We propose novel, high-precision explicit equations for D (Eqs. 56 and 60). These achieve maximum relative errors of 0.017 % and 0.0085 %, respectively. We also introduce an innovative friction factor equation (54) with 0.086 % error. Validated across the entire Moody diagram ($\varepsilon/D = 0$ to 0.05, and $2300 \leq R \leq 10^8$) using a brute-force approach with over 7 million data points, these non-iterative solutions outperform existing models. A comprehensive set of statistical metrics including Mean Absolute Error (MAE), Root Mean Square Error (RMSE), correlation coefficients (R^2 and Pearson's R), Bias, Mean Relative Error (MRE), Standard Deviation (SD), Coefficient of Variation (CV), and maximum relative error were employed to assess the accuracy and reliability of the proposed and existing formulas; the results of the Statistical metrics confirm their robustness, establishing a new benchmark for accuracy in pipeline design. This advancement enhances efficiency and reliability in water, oil, and gas transport systems.

Introduction

Turbulent flow is the most common type encountered in water distribution networks. It involves random variations in fluid particle trajectories over time. Flow in pressurized pipes follows this functional relationship:

$$F(Q, S_f, D, \varepsilon, \nu) = 0 \tag{1}$$

Here, Q is the discharge, S_f is the head loss per unit pipe length, D is the pipe diameter, ε *is* the equivalent sandgrain roughness height, ν is the kinematic viscosity of the flowing fluid.

Among these parameters influencing the flow, only Q, D, and S_f are of practical interest. Turbulent flow in a pipe is modeled by the universally known Darcy-Weisbach formula [1], where the friction factor f is evaluated using the Colebrook-White equation [2,3].

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Mathematical Symbols and Abbreviations

D	Pipe diameter
D_{rm}	Pipe diameter for modified rough model
D_r	Pipe diameter for rough model
D_1	Pipe diameter for rough model
D_2	Diameter for Smooth Regime
D_3	Diameter for Transition Zone
f	Friction factor
f_{rm}	Friction factor for modified rough model
f_r	Friction factor for rough model
g	acceleration of gravity
ΔH	Head loss
S_f	Head loss per unit pipe length
ε	Equivalent sandgrain roughness height
ε_{rm}	Equivalent sandgrain roughness height for modified rough model
MAE	Mean Absolute Error
Willmott's d	Willmott's Index of Agreement
Pearson's R	Pearson's Correlation Coefficient
SD	Standard Deviation
ε_r	Equivalent sandgrain roughness height for rough model
Q	Discharge
Q_{rm}	Discharge in modified rough model
R	Pipe Reynolds number
R_{rm}	Reynolds number for modified flow model
R_r	Reynolds number for rough model
T	Temperature
ν	Kinematic viscosity of the flowing fluid
τ	Correction coefficient for rough model
y_1, y_2, γ	Dimensionless parameter
υ^*	the non-dimensional viscosity
ε^*	the non-dimensional roughness
D^*	The non-dimensional diameter
MRMM	Modified Rough Model Method
RMSE	Root Mean Square Error
R^2	Coefficient of Determination
MRE	Mean Relative Error
CV	Coefficient of Variation

These relationships are written as follows:

$$S_f = \frac{8fQ^2}{\pi^2 g D^5} \tag{2}$$

$$\frac{1}{\sqrt{f}} = -2log\left[\frac{\varepsilon/D}{3,7} + \frac{2,51}{R\sqrt{f}}\right] \tag{3}$$

Where:

R: the pipe Reynolds number,

V: mean velocity.

g: gravitational.

The Reynolds number is given by the Eq. (4):

$$R = \frac{4Q}{\pi D \nu} \tag{4}$$

Where:

ν: The kinematic viscosity.

The flow is considered turbulent if the Reynolds number is $R \geq 2300$. Using Eqs. (2), (3), and (4), the diameter in a turbulent flow pipe can be calculated using Eq. (5) as follows:

$$D = \left(\frac{8Q^2}{\pi^2 g\ s_f}\right)^{1/5}\left(-2log\left[\frac{\varepsilon/D}{3,7}+\frac{2,51}{R\sqrt{f}}\right]\right)^{-2/5} \tag{5}$$

Determining the diameter in pipe flow problems typically requires the use of diagrams or iterative methods. The main difficulty in obtaining a direct solution lies in the implicit form of the Colebrook-White equation. In general, these solutions are often inaccurate, limited, or complex, and can be time-consuming. For this reason, many authors have proposed approximate solutions to the Colebrook-White equation based on the specific case studied, such as Moody [4], Achour and Bedjaoui [5], and Zeghadnia et al. [6–8], among others.

Eq. (5) can be developed and rewritten as a function of the known flow parameters Q, S_f, ε *and* ν follows:

$$D = \left(\frac{8Q^2}{\pi^2 g\ s_f}\right)^{1/5}\left(-2log\left[\frac{\varepsilon/D}{3,7}+\frac{2,51\sqrt{2}\nu}{2\sqrt{gs_f D^3}}\right]\right)^{-2/5} \tag{6}$$

All parameters governing turbulent flow are consolidated in Eq. (6), which is expressed as a function of$Q$, S_f, ε *and* ν; This formulation establishes Eq. (6) as the appropriate reference for all subsequent processes. Derived from fundamental principles such as the Darcy-Weisbach, Colebrook-White, and Reynolds equations, Eq. (6) is widely accepted as a baseline in similar studies [5,9,10]. The Eq. (6) is implicit and requires an iterative solution.

Providing an explicit equation for designing the diameter of a turbulent flow pipe has attracted the attention of several authors. Numerous papers have been written on this topic. For instance, Advaui et al. [11] studied the link between flow regimes, pipe roughness, and Reynolds number, extending Prandtl-Karman's equations to rough pipes and proposing a simplified method for pipe diameter estimation in turbulent flow. Rajaratnam [12] offered an analytical solution for diameter in fully rough turbulent flow. It applies only to that regime. For broader conditions, they used the Colebrook-White transition function, enhancing versatility in circular pipes, while Hager [13] related the coefficient k to absolute roughness ε. Swamee and Rathie [14] refined friction factor formulas, and Swamee and Swamee [15] introduced general equations for diameter and discharge across laminar, transitional, and turbulent flows. Achour and Bedjaoui [5] applied a constant Colebrook-White friction factor with correction terms for direct pipe-flow calculations, whereas Babajimopoulos and Terzidis [16] provided explicit equations to avoid iterative solutions. Medina et al. [17] proposed an empirical model slightly outperforming Swamee-Jain, while Yetilmezsoy et al. [18] developed a hybrid regression-based formula tested on 10,000 scenarios, The authors report that the model was tested against 10,000 scenarios, showing significant improvements in prediction accuracy and uncertainty reduction compared to existing models. Lamri and Easa [9] introduced explicit Lambert W-function solutions and uncertainty-based design graphs. Recently, Brkić et al. [19] used symbolic regression for diameter solutions. The authors utilized tools like PySR and Eureqa, and explored brute-force, Lambert-transformed, and dimensionless approaches. Models were proposed to reduce iteration and achieving lower errors than nomograms

This literature covers innovative pipe diameter and flow regime studies. Many focus on alternatives to Colebrook's challenges. Iterative methods were complex, leading to direct solutions via Lambert W, regression, and groups. Models offer higher accuracy and benefits. Yet, limitations exist; selections depend on conditions. These contribute to design, suggesting future generalizable research.

This study's objective is to enhance explicit diameter formulas' precision and reliability. They apply broadly while staying simple. Metrics like Mean Absolute Error (MAE), Root Mean Square Error (RMSE), correlation coefficients (R^2 and Pearson's R), Bias, and Coefficient of Variation (CV), evaluate against models, showing superiority.

The paper reviews key formulas and their accuracy. It develops an innovative MRMM formula with exceptional precision over the Moody diagram ($0 \le \varepsilon/D \le 0.05,\ 2300 \le R \le 10^8$), This provides versatile solutions, establishing a new benchmark for precision in explicit friction factor and pipe diameter calculations.

Methodology

Review of existing formulas

Numerous authors, including Genereaux [20], Dubin [21], Peters and Timmerhaus [22,23], Nolte [24], and Capps [25], have published formulas and nomographs for estimating pipe diameter for economic purposes without considering the type of flow. Thresh [26], Dupuit [27], Swamee and Sharma [28], and Garg [29] suggested that the pipe diameter can be represented by the following equation [30,31]:

$$D = K\sqrt{Q} \tag{7}$$

where: k is a parameter, and Q represents the required flow rate.

Thresh [26] suggested: $1,3 \le k \le 1,46$ [28]. Dupuit suggested a value of: $k = 1,60$, while Garg [29] proposed a range of: $0,97 \le k \le 1,22$ [31].

Additional researchers have utilized the Manning-Stickler model to develop explicit formulas. Notable examples include the work of Zeghadnia [7,32–35] which addresses various scenarios, including fully filled circular pipes.

Other authors, using the Colebrook-White equation, have proposed explicit solutions for the third problem. Given Eqs. (2) and (3), an iterative solution is inevitable due to the implicit nature of the Colebrook-White resistance flow equation. To address this challenge, several researchers have worked on making the solution easier and more direct [10], including:

1. Advaui R. M [11]:

A direct analytical solution for the diameter of a pipe in fully developed rough turbulent flow was proposed by Advani [11] using the following formula:

$$D = \left(\frac{Q^2\left(\varepsilon^{1/3}\right)}{27\ S_f}\right)^{3/16} \tag{8}$$

The recommended range for Eq. (8) is:
$R \rightarrow \infty$ and $10^{-5} \leq \varepsilon/D \leq 5.10^{-2}$

2. Rajaratnam N [12]

In 1960, Rajaratnam presented a direct analytical solution for calculating the diameter of a pipe in fully developed rough turbulent flow. Given $Q, S_f, \varepsilon,$ *and*
ν, the diameter of the pipe can be easily determined by following these three steps:
Step 01: From the Darcy-Weisbach equation, compute the dimensionless quantity γ, where:

$$\gamma = \frac{S_f}{Q^2} g\varepsilon^5 \tag{9}$$

Step 02: Next, estimate the relative roughness using Eq. (10):

$$\varepsilon \Big/ D = \left(\frac{\gamma}{0.1178}\right)^{1/5.297} \tag{10}$$

Step 03: Finally, obtain the pipe diameter using the following equation:

$$D = \varepsilon(\varepsilon/D)^{-1} \tag{11}$$

Equations (9), (10), and (11) are applicable only to fully developed turbulent flow in rough pipes.
$R \rightarrow \infty$ and $10^{-5} \leq \varepsilon/D \leq 5.10^{-2}$

3. Swamee and Jain [36]:

According to Swamee and Jain, the solution to the third problem can be achieved in a few steps. The known variables $Q,\ S_f,\ \varepsilon$, and ν help determine the following set of equations:
Step 01: In the first step, compute the non-dimensional viscosity υ^*:

$$\upsilon^* = \nu\left(\frac{1}{gS_fQ^3}\right)^{1/5} \tag{12}$$

Step 02: In the second step, the non-dimensional roughness ε^* can be computed using the known variables:

$$\varepsilon^* = \varepsilon\left(\frac{gS_f}{Q^2}\right)^{1/5} \tag{13}$$

Step 03: The non-dimensional diameter D^* can be determined using the following approximation, where all parameters are known:

$$D^* = 0,66\left(\varepsilon_*^{1,25} + \nu_*\right)^{0,04} \tag{14}$$

Step 04: Finally, the pipe diameter can be obtained as follows:

$$D = D^*\left(\frac{Q^2}{gS_f}\right)^{1/5} \tag{15}$$

The authors recommend the proposed equations for the following range:
$2.10^{-6} \leq \varepsilon/D \leq 2.\ 10^{-2}$ and $3\ 000 \leq R \leq 3.10^8$

4. Hager [13]

Hager's approach involves evaluating the diameter D for each part of the turbulent flow range. This approach was developed based on concepts proposed by Swamee and Jain [36,5,13,10].

4.1. Case of Smooth Turbulent Flow:

In the smooth domain, corresponding to $\varepsilon/D = 0$, the flow resistance does not depend on the relative roughness. In this case, two steps are recommended to compute the pipe diameter:

Step 01: First, evaluate the non-dimensional viscosity using Eq. (12),

Step 02: Next, to determine the diameter from the non-dimensional diameter D^*, Hager suggests the following solutions: using Eq. (16):

$$D = D^*\left(\frac{Q^2}{gS_f}\right)^{1/5}, \ D^* = \frac{2}{5}log\left[\frac{-54,64}{log(\upsilon^*)}\right] \tag{16}$$

Hager recommended that the (16) can be applied in the range [5]:

$10^{-9} < \nu^* < 10^{-3}$ and $2450 < R < 4.10^9$

4.2. Case of Turbulent Flow in the Completely Rough Regime:

In the rough turbulent domain, the effect of the Reynolds number R on flow characteristics, such as the friction coefficient f and energy gradient S_f, is not significant. This corresponds to: $\nu\to 0, \ R\to\infty$. From the known values of the parameters Q, S_f, ε and ν, the diameter can be computed as follows:

Step 01: Compute the non-dimensional viscosity υ^* using Eq. (12).

Step 02: In the second step, calculate the value of the non-dimensional roughness ε^* by applying Eq. (13).

Step 03: To calculate the diameter using the non-dimensional diameter D^*, Hager proposes the following solutions:

$$D = D^*\left(\frac{Q^2}{gS_f}\right)^{1/5}, D^* = \frac{\varepsilon^{*0.03}}{1,853}, for\ 10^{-8} < \varepsilon \Big/ D < 10^{-4} \tag{17}$$

$$D = D^*\left(\frac{Q^2}{gS_f}\right)^{1/5}, D^* = \frac{\varepsilon^{*1/16}}{1,422}, \ for\ 7.10^{-4} < \varepsilon \Big/ D < 7.10^{-2} \tag{18}$$

5. Swamee and Rathie [14]

Using Lagrange's implicit equation theorem, Swamee and Rathie [14] presented an equation for the diameter problem, which is expressed as follows:

$$D = \left(\frac{Q^2}{gS_f}\right)^{1/5}\left[\varphi + 0.9647ln\left(\frac{0.1518\varepsilon_*}{\nu_*}\varphi^{2/5} + \varphi^{3/5}\right) \times \left[-1 + \frac{0.5788\left(0.1012\varepsilon_* + \nu_*\varphi^{1/5}\right)}{(0.1518\varepsilon_* + \nu_*\varphi^{1/5})\varphi}\right]\right]^{-2/5} \tag{19}$$

Where:

$$\varphi = 0,9647ln\left[\frac{1}{1,78\nu_*}\right] \tag{20}$$

The non-dimensional parameters, viscosity $\nu_*$and roughnessε_*, can be calculated using Eqs. (12) and (13), respectively.

The recommended range is defined as:

$3.10^3 \le R \le 3.10^8$ and $2.10^{-6} \le \varepsilon/D \le 5.10^{-2}$

6. Swamee and Swamee [15]

In this work, the authors proposed a general equation covering laminar, transitional, and turbulent flows:

$$D = 0,66\left[\left(214,75\frac{\nu Q}{gS_f}\right)^{6,25} + \varepsilon^{1,25}\left(\frac{Q^2}{gS_f}\right)^{4,75} + \nu Q^{9,4}\left(\frac{1}{gS_f}\right)^{5,2}\right]^{0.04} \tag{21}$$

Eq. (21) is valid for the following range:

$2.10^{-6} \le \varepsilon/D \le 2.10^{-2}$ and $0 \le R \le 3.10^8$

7. Achour and Bedjaoui [5]

Based on Rough Model Method (RMM), with reference $\varepsilon_r/D_r = 0,037$. The subscript 'r' refers to the RMM. For $R_r \to\infty$, using Eq. (3), this involves $f_r = 1/16$.

Given Q, ε, S_f and ν, the diameter can be computed using Eq. (22):

$$D = D_r\left[-\frac{1}{2}log\left(\frac{\varepsilon/D_r}{3,7\tau}+\frac{10,04}{R_r\tau^{3/2}}\right)\right]^{-2/5} \tag{22}$$

The diameter D_r, Reynolds number R_r and the correction coefficient τ are the parameters of Rough Model, they can be computed directly using the Eqs. (23), (24) and (25) respectively:

$$D_r = \left(2\pi^2\right)^{-1/5}\left(\frac{Q^2}{gS_f}\right)^{1/5} \tag{23}$$

$$R_r = \frac{4Q}{\pi D_r \nu} \tag{24}$$

$$\tau = 1,35\left[-log\left(\frac{\varepsilon/D_r}{4,75}+\frac{8,5}{R_r}\right)\right]^{-2/5} \tag{25}$$

The Eq. (22) is valid for:
$0 \le \varepsilon/D \le 5.10^{-2}$ and $R > 2300$

8. Babajimopoulos and Terzidis [16]

The authors developed an explicit equation to solve the pipe diameter problem as follows:

$$D = \left(\frac{8Q^2}{\pi^2 g s_f}\right)^{1/5}\left[-1.997log\left[0.3523t\left|log\left(0.3055T^{1.007}+\frac{2.803}{K^{0.995}}\right)\right|^{0.4}+\left(\frac{2.688}{K}\left|log\left(0.3055T^{1.007}+\frac{2.803}{K^{0.995}}\right)\right|^{0.6}\right)\right]\right]^{-2/5} \tag{26}$$

where:

$$T = \frac{\varepsilon\left(9,81\ S_f\right)^{0.2}}{Q^{0,4}} \tag{27}$$

$$k = \frac{q^{0,6}\left(9,81\ S_f\right)^{0,2}}{\nu} \tag{28}$$

Eq. (26) is applicable for:
$0 \le \varepsilon/D \le 5.10^{-2}$ and $4000 \le R \le 10^8$

9. Medina et al. [17]

Using a regression analysis method, an explicit equation for the pipe diameter was proposed, as shown in Eq. (29):

$$D = \left[\sqrt{\frac{\varepsilon Q^{7,6}}{g^{3,8}}}\left(\frac{0,11}{S_f}\right)^{1,9}+\frac{\nu^{0,4}Q^{3,76}}{g^{2,08}}\left(\frac{0,13}{S_f}\right)^{2,08}\right]^{0,1} \tag{29}$$

The Eq. (29) is recommended for the range:
$10^{-6} \le \varepsilon/D \le 5.10^{-2}$ and $3000 \le R \le 3,2.10^8$

10. Yetilmezsoy et al. [18]

Based on a comprehensive hybrid programming-based methodology, Eq. (30) was developed to calculate the pipe diameter as follows:

$$D = 0,2985\left(\varepsilon^{0,05451}\right)\times T^{-0,0002713}\times L^{0,1875}\times Q^{0,3743}\times \Delta H^{-0,1879} \tag{30}$$

Where:
ΔH: head loss,
L : Pipe length,
T: is the water temperature.
The Eq. (30) can be rewritten as function of "S_f" as follows:

$$D = 0,2985\left(\varepsilon^{0,05451}\right)\times T^{-0,0002713}\times Q^{0,3743}\times {S_f}^{-0,1879}\times L^{-0.0004} \tag{31}$$

For unit length, Eq. (32) can be converted into Eq. (33):

$$D = 0,2985\left(\varepsilon^{0,05451}\right)\times T^{-0,0002713}\times Q^{0,3743}\times {S_f}^{-0,1879} \tag{32}$$

The Eqs. (31) and (32) are recommended for the following range:
$1,42.10^{-6} \le \varepsilon/D \le 0,135$, $7,68.10^{4} \le R \le 1,78.10^{7}$ and
$5^{\circ}C \le T \le 100^{\circ}C$, $5m \le L \le 100m$

11. Lamri and Easa [9]

Using the Lambert W-function, the authors propose using Eqs. (33), (34), and (35), each of which is intended for a specific flow regime: smooth, rough, and transitional, respectively:

11.1. Rough Regime

$$D_1 = \left(\frac{Q^2}{gS_f}\right)^{1/5}\left[2\pi\frac{ln(y_1) - ln(ln(y_1)) + ln(ln(y_1))/ln(y_1)}{5\sqrt{2}ln(10)}\right]^{-2/5} \tag{33}$$

Valid for:
$3,42.10^{-7} \le \varepsilon^* \le 2,83.10^{-2}$ and $0,342 \le D_1 \le 0,566$
Or:
$6,04.10^{-7} \le \varepsilon/D \le 8,27.10^{-2}$ and $R \to \infty$

11.2. Smooth Regime

$$D_2 = \left(\frac{Q^2}{gS_f}\right)^{1/5}\left[3\pi\frac{ln(y_2) - ln(ln(y_2)) + ln(ln(y_2))/ln(y_2)}{5\sqrt{2}ln(10)}\right]^{-2/5} \tag{34}$$

Applicable for the following range:
$1,267.10^{-8} \le \nu^* \le 8,28.10^{-4}$ and $0,335 \le D_2 \le 0,512$
The limits can be expressed as a function of:
$3005 \le R \le 3.10^{8}$ and $\varepsilon/D = 0$

11.3. Transition Zone

$$D_3 = 1,0157\left({D_1}^{17,4} + 1,78{D_2}^{18,85}\right)^{0.09}\left({D_1}^{21,04} + 10{D_2}^{26,5}\right)^{-0,026} \tag{35}$$

The authors recommend the following range:
$0,45.10^{-7} \le \varepsilon^* \le 2,88.10^{-2}$, $7,5.10^{-9} \le \nu^* \le 8.28.10^{-4}$ and $0,3453 \le D_3 \le 0,577$
Or, as a function of the Reynolds number and relative roughness, as follows:
$2,67.10^{3} \le \mathrm{R} \le 4,92.10^{8}$ and $7,810^{-8} \le \varepsilon/D \le 8,34.10^{-2}$
Where:

$$y_1 = \frac{5\sqrt{2}ln(10)}{2\pi}\left(\frac{3,7}{\varepsilon^*}\right)^{5/2} \tag{36}$$

$$y_2 = \frac{5\sqrt{2}ln(10)}{3\pi}\left(\frac{\sqrt{2}}{2,51\nu^*}\right)^{5/3} \tag{37}$$

12. Dejan Brkić et al. [19]

The authors provided an approximate equation as a solution for the unknown pipe diameter D using the Symbolic Regression method, where the most accurate result was the following:

$$D = \sqrt{0,0248Q + sin(0,405Q + 0,534\varepsilon) + QA + \frac{0,00736Q + sin(0,247QA) + 0,00615A + 0,0000874}{\sqrt{S_f}}} \tag{38}$$

Where:

$$A = \sqrt{0,247\varepsilon} \tag{39}$$

The validity for the range proposed by the authors is as follows:
$3.10^{-7} \leq \varepsilon/D \leq 5.10^{-2}$ and $R \geq 2320$

Accuracy indices measurement

To determine which of these formulas is the most accurate, two primary approaches exist: one based on the maximum error and another using a global statistical analysis (mean error, standard deviation, etc.). The maximum-error approach ensures insight into the worst possible deviation from the reference solution [5,19,15,14]; Specifically:

- ➢ Reference Formula: The implicit Eq. (6) is taken as the reference formula for evaluating the accuracy of the approximate solutions.
- ➢ Validity Range: The accuracy of each formula is analyzed within its stated valid range.
- ➢ Maximum Deviation: The maximum deviation (in percentage) is estimated using Eq. (40):

$$\frac{\Delta D}{D} = \left|\frac{D_{eq(06)} - D_{approximation}}{D_{eq(06)}}\right| 100\% \tag{40}$$

- ➢ Simplicity Criterion: As a fourth criterion, the simplicity of the proposed formula is evaluated.
- ➢ Extensive Data Calculation: To thoroughly assess each approximate solution's accuracy, more than seven and a half million (7.5 million) values are computed.
- ➢ Traditionally, flow is deemed laminar when $Re < 2300$. However, numerous authors argue that the Colebrook-White equation remains valid for $Re \geq 2300$, on the grounds that it captures the gradual transition from smooth turbulent flow to fully rough, turbulent flow. Consequently, it can be applied as soon as one moves beyond the laminar regime.

By reference to certain experimental curves (notably the Moody chart), the equation's applicability has been extended down to $Re = 2300$. In effect, Colebrook is regarded as valid across the entire non-laminar domain depicted on the Moody chart [37] or within a similar range [5,19,38–48]

Meanwhile, the global statistical approach provides a comprehensive overview of each formula's performance across a broad spectrum of flow conditions ([17,12,18]; Lamri and Easa [9]. In the comparative analysis of theoretical and computed diameters and to evaluate the maximum relative error nine benchmark statistical parameters are employed: Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Coefficient of Determination (R^2), Pearson's Correlation Coefficient (R), Bias Coefficient, Mean Relative Error (MRE), Standard Deviation (SD), Coefficient of Variation (CV), and Willmott's Index of Agreement (d).

To better capture both the reliability and the robustness of these explicit equations, both methodologies—maximum-error and global statistical—are applied in tandem.

Table 1
Maximum relative errors of approximate solutions.

Author		**Reynolds Number**	**Relative Roughness**	**Max *Error* ($\Delta D/D$) %**
Advaui [11]		$R \to \infty$	$10^{-5} \leq \varepsilon/D \leq 5.10^{-2}$	30,04%
Rajaratnam [12]		$R \to \infty$	$10^{-5} \leq \varepsilon/D \leq 5.10^{-2}$	9,51%
Swamee and Jain [36]		$3\,000 \leq R \leq 3.10^8$	$2.10^{-6} \leq \varepsilon/D \leq 2.10^{-2}$	4,00%
Hager [13]	Smooth turbulent flow	$2450 < R < 4.10^9$	$\varepsilon/D = 0$	2,619
	Completely Rough regime	$R \to \infty$	$10^{-8} \leq \varepsilon/D \leq 10^{-4}$....(Eq. 17)	2,62%
		$R \to \infty$	$7.10^{-4} \leq \varepsilon/D \leq 7.10^{-2}$.....(Eq. 18)	1,4%
Swamee and Rathie [14]		$3\,000 \leq R \leq 3.\,10^8$	$10^{-6} \leq \varepsilon/D \leq 5.10^{-2}$	2,16%
Swamee and Swamee [15]		$0 \leq R \leq 3.10^8$	$2.10^{-6} \leq \varepsilon/D \leq 2.10^{-2}$	4,00%
Achour and Bedjaoui [5]		$R > 2300$	$0 \leq \varepsilon/D \leq 5.10^{-2}$	0,042%
Babajimopoulos and Terzidis [16]		$4000 \leq R \leq 4.10^8$	$10^{-5} \leq \varepsilon/D \leq 5.10^{-2}$	0,061%
Medina et al. [17]		$3000 \leq R \leq 3,2.10^8$	$10^{-6} \leq \varepsilon/D \leq 5.10^{-2}$	4,00%
Yetilmezsoy et al. [18]		$7,68.10^4 \leq R \leq 1,78.10^7$	$1,42.10^{-6} \leq \varepsilon/D \leq 0,135$ And $5^\circ C \leq T \leq 100^\circ C$ $5m \leq L \leq 100m$	49,86%
Lamri and Easa [9]	Smooth Regime	$3005 \leq R \leq 3.10^8$	$\varepsilon/D = 0$	0,016 %
	Rough Regime	$R \to \infty$	$6,04.10^{-7} \leq \varepsilon/D \leq 8,27.10^{-2}$	0,013 %
	Transition Zone	$2,67.10^3 \leq R \leq 4,92.10^8$	$7,810^{-8} \leq \varepsilon/D \leq 8,34.10^{-2}$	2,641%
Brkı́ć et al. [19]		$R \geq 2320$	$3.10^{-7} \leq \varepsilon/D \leq 5.10^{-2}$	100%
Proposed Formulas	Entire range of Moody diagram	$R \geq 2300$	$0 \leq \varepsilon/D \leq 5.10^{-2}$	0,017 %
	Smooth regime	$R \geq 2300$	$\varepsilon/D = 0$	0,008%

Caption: Table 1 shows maximum relative errors (%) for approximate solutions compared to implicit Eq. (6), within stated validity ranges.

Discussion and analysis of the accuracy of the approximate solutions

Before introducing the new equations, evaluate each approximate solution from the literature by maximum recorded error. Analysis criteria applied to 12 equations. Table 1 summarizes maximum errors for each reviewed formula.

From Table 1, four Eqs. (8), (30), and (38) show high deviations compared to Eq. (6). Maximum error reaches 100 % for Eq. (38), as shown in Fig. 1. Only Eq. (8) recommended for rough turbulent flow.

Of the twelve (12) formulas, eight (08) exhibited maximum relative errors ranging from 9,51 % to 1,4 %, specifically for Eqs. (11), (15, 16, 17, 18, 19, 21, 29) and (35) (refer to Figs. 2 and 3). With the exception of Eq. (11), the remaining formulas maintained a maximum relative error above 4 %.

Eqs. (8), (11, 17, 18), and (33) recommended only for rough turbulent flow. From Table 1, the difference between them and Eq. (33) is significant. Eq. (17) addresses only a limited range of relative roughness. Its maximum deviation is greater than that of Eqs. (18) and (33), as illustrated in Fig. 3.

The most accurate formulas remain in the set, which will be compared to the proposed formulas. This group consists of Eqs. (22), (26, 33), and (34). The maximum deviation is 0,061 % ($6{,}1.10^{-2}$%) for Eq. (26), while it is 0,042 % ($4{,}2.10^{-2}$%)for Eq. (22), as shown in Fig. 4. Eqs. (22) and (26) cover the entire range of the Moody diagram; however, Eqs. (33) and (34) are recommended only for rough and smooth turbulent flow, respectively, as reported in Table 1.

According to the statistical analysis results in Table 2, the formulas proposed by Advaui [11], Yetilmezsoy et al. [18], and Brkić et al. [19] exhibit somewhat higher error margins with MAE values reaching up to 0.91 and RMSE values of 1.05. The outputs from these models also demonstrate some bias and higher Mean Relative Error (MRE) values (21.79 % and 34.95 %, respectively), which may limit their effectiveness in applications requiring very precise calculations. Additionally, Brkić's formula presents a negative R^2 (−0.90), which suggests some potential challenges in terms of predictive accuracy. These results point to the possible limitations of these models in precise applications and suggest that careful consideration is needed when applying them.

In contrast, the formulas of Hager [13], Medina et al. [17], and Swamee and Rathie [14] exhibit moderate accuracy, with MAE values between 0,007 and 0,064 and corresponding RMSE values. Hager's formula performs well in smooth turbulent flow but demonstrates increased errors and variability in completely rough regimes. Medina et al. achieve strong correlations ($R^2 = 0{,}99$) and a low MRE (2,61 %), though variability (CV = 53,50) remains a limitation. Swamee and Rathie provide moderate precision, with an MRE of 1,34 %, making their formula appropriate for scenarios where extreme precision is not critical.

Another set of formulas includes those proposed by Swamee and Jain [36], Swamee and Swamee [15], and Rajaratnam N [12]. These formulas strike a balance between accuracy and reliability, with MAE values ranging from 0,025 to 0,043 and RMSE values from 0,032 to 0,082. Swamee and Jain achieve excellent correlations ($R^2 = 0.99$), and their low Bias makes them robust and reliable for general applications. Swamee and Swamee demonstrate improved accuracy and adaptability, although variability (CV = 70,68) slightly reduces their stability under extreme conditions. Rajaratnam's formula offers moderate precision and strong correlations but is comparatively less effective than Swamee and Jain's.

Finally, the proposed formulas, along with those by Bachir Achour and Bedjaoui [5] and Lamri and Easa [9] (in the smooth regime), exhibit exceptional precision. With MAE and RMSE values as low as 0,0001, these methods excel across a wide range of conditions, including the entire Moody diagram. The proposed formulas achieve perfect R^2 and minimal variability (CV between 60 and 74),

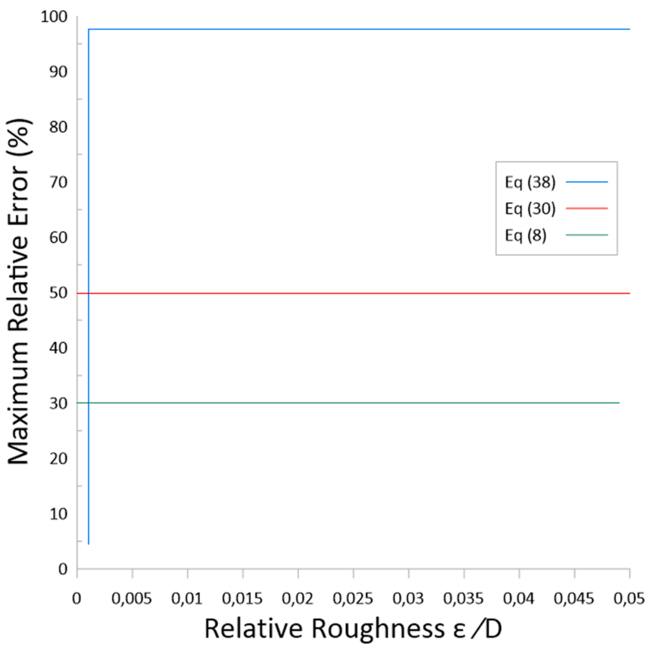

Fig. 1. Maximum relative errors (%) for Eqs. (8), (30), and (38).

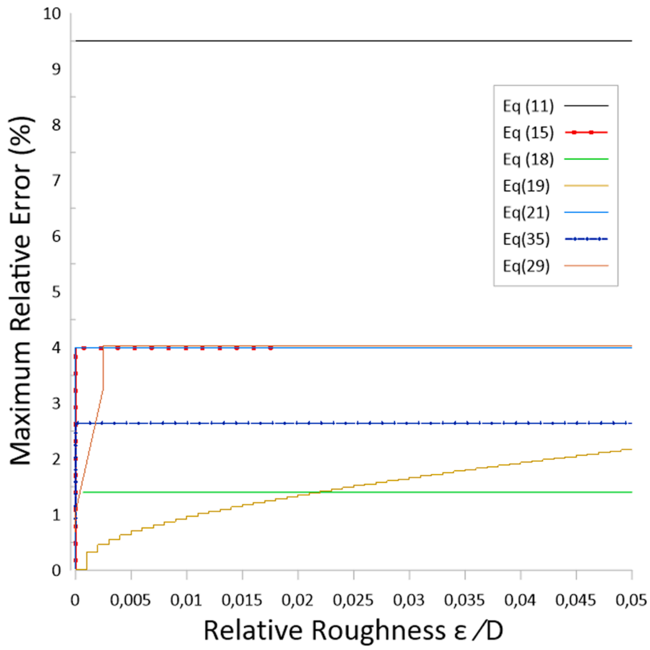

Fig. 2. Maximum relative errors (%) for Eqs. (11), (15, 18, 19, 21, 29) and (35).

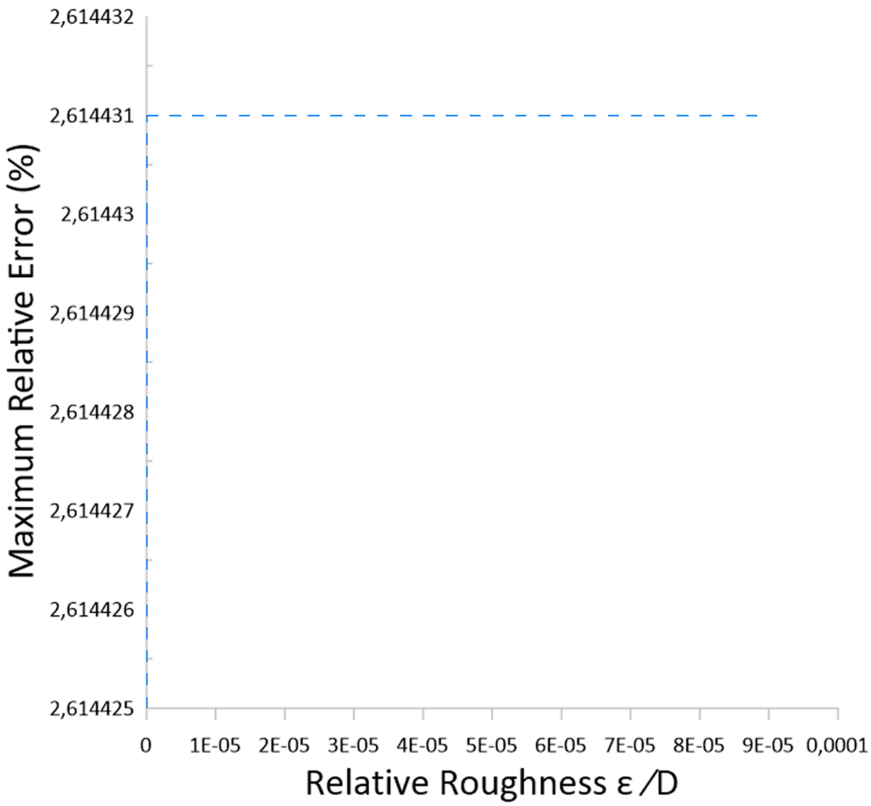

Fig. 3. Maximum relative errors (%) for Eq. (17). For: R→∞ and $10^{-8} \leq \varepsilon/D \leq 10^{-4}$.

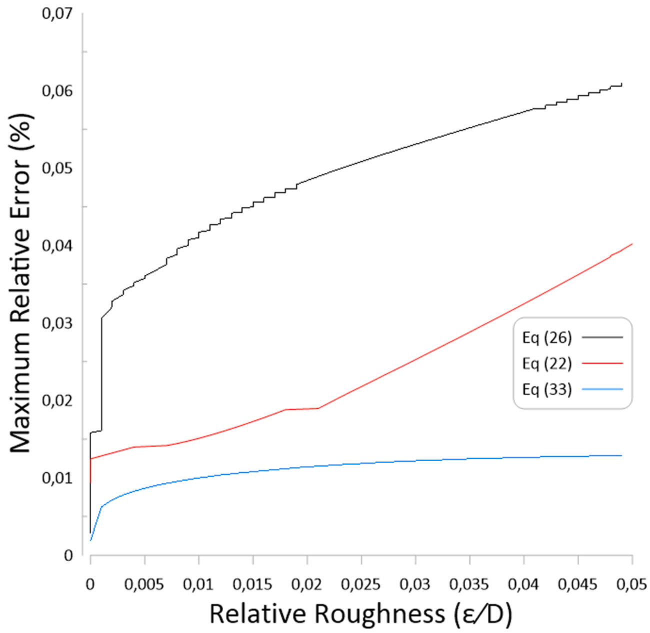

Fig. 4. Maximum relative errors (%) for Eqs. (22) and (26).

Table 2
Statistical performance metrics of hydraulic flow formulas.

Author		MAE	RMSE	R^2	Pearson's R	Bias	MRE (%)	SD	CV	Willmott's d
Advaui [11]		0,686	0,756	0,78	0,99	−0,686	21,79	1,29	50,41	0,94
Rajaratnam [12]		0,043	0,082	0,99	0,99	−0,043	2,51	1,04	45,19	0,99
Swamee and Jain [36]		0,031	0,039	0,99	0,99	−0,021	2,12	0,85	60,31	0,99
Hager [13]	Smooth turbulent flow	0,007	0,009	0,99	01	0,007	0,95	0,65	83,78	01
	Completely Rough regime…. Eq. (17)	0,012	0,014	0,99	01	0,012	1,36	0,73	61	0,99
	Completely Rough regime…. Eq. (18)	0,014	0,017	0,99	01	- 0,01	0,65	1,38	54,92	01
Swamee and Rathie [14]		0,035	0,043	0,99	01	0,035	1,34	1,22	61,54	0,99
Swamee and Swamee [15]		0,025	0,032	0,99	0,99	−0,02	1,96	0,90	70,68	0,99
Achour and Bedjaoui [5]		0,0001	0,0001	01	01	−0,0001	0,010	0,79	74,87	01
Babajimopoulos and Terzidis [16]		0,0006	0,0009	01	01	−0,0006	0,040	1,082	78,34	01
Medina et al. [17]		0,064	0,077	0,99	0,99	−0,064	2,61	1,066	53,50	0,99
Yetilmezsoy et al. [18]		0,55	0,78	0,76	0,99	−0,55	34,95	1,04	99,94	0,92
Lamri and Easa [9]	Smooth Regime	0,0001	0,0001	01	01	0,0001	0,006	0,75	55,65	01
	Rough Regime	0,0003	0,0004	01	01	0,0003	0,01	1,29	51,73	01
	Transition Zone	0,006	0,007	0,98	0,99	0,005	1,10	0,07	13,97	0,99
Brkíc et al. [19]		0,91	1,05	−0,90	0,67	0,515	54,07	1,23	55,82	0,7
Proposed Formulas	Entire range of Moody diagram	0,0001	0,0001	01	01	0,0001	0,007	0,81	60,18	01
	Smooth regime	0,00	0,00	01	01	0,00	0,002	0,76	73,81	01

Caption: Table 2 presents statistical metrics (MAE, RMSE, R^2, etc.) for hydraulic flow formulas across turbulent regimes.

making them highly reliable and versatile. Similarly, Bachir Achour and Bedjaoui's formulas demonstrate near-zero Bias and strong performance metrics, establishing them as recommended options for applications demanding extreme accuracy. Lamri and Easa [9] also produce outstanding results in the smooth regime, though their performance slightly diminishes in the transition zone.

New equations using the modified rough model method (MRMM)

The reference roughness model compares any real flow to a "standard" flow with a fixed and well-defined roughness. To bridge the gap between this simplified model and the actual flow conditions, correction coefficients are introduced, reconciling the implicit equations describing the real flow with the explicit equations of the reference model. In the so-called "Rough Method" (RM), for example, a relative roughness of, $\varepsilon_r/D_r = 0{,}037$, and a friction factor equal to: $f_r = \frac{1}{16}$ (which can be derived using Eq. (3) in the limit $R\rightarrow\infty$) are adopted. All subsequent formulas rely on these reference values, enabling a consistent comparison and unified calibration between the ideal flow in the reference roughness model and the real flow in various hydraulic systems [5,10].

To improve the solutions, the model can be adjusted by using a new, appropriate value for the relative roughness. Through the trial-and-error method, the optimal value of relative roughness for solving the pipe diameter problem is as follows:

$$\frac{\varepsilon_{rm}}{D_{rm}} = \frac{258,57}{1000} \tag{41}$$

The subscript 'rm' is used to represent the Modified Rough Model (MRM). The newly chosen value for relative roughness provides the updated details of the MRM, which corresponds to a fictitious pipe.

Based on Eq. (3), and for: $R \rightarrow \infty$, the new value of the friction factor for the MRM is given by Eq. (42):

$$f_{rm} = 0,1872 \tag{42}$$

Using Eqs. (2) and (42), we obtain the new formula for the diameter of the Modified Rough Model (MRM):

$$D_{rm} = \left[\frac{1,4976 Q_{rm}^2}{g\pi^2 S_{f\,rm}}\right]^{1/5} \tag{43}$$

The Reynolds number for the MRM can be expressed using Eq. (44):

$$R_{rm} = \frac{4Q_{rm}}{\pi \nu_{rm} D_{rm}} \tag{44}$$

By substituting Eq. (43) into Eq. (44), the latter can be rewritten as a function of Q_{rm}, ν_{rm} *and* $S_{f\,rm}$:

$$R_{rm} = \left(\frac{4}{\nu_{rm}}\right)\left(\frac{Q_{rm}^3 g S_{f\,rm}}{1,4976\pi^3}\right)^{1/5} \tag{45}$$

Based on the following assumptions: $Q_{rm} = Q$, $S_{f\,rm} = S_f$, $\varepsilon_{rm} = \varepsilon$ and $\nu_{rm} = \nu$; from Eq. (2), we can deduce the relationship between the MRM and the model under study as follows:

- The relationship between D_{rm} and D:

$$D = D_{rm}\left[\frac{f}{f_{rm}}\right]^{1/5} \tag{46}$$

Using Eq. (42), Eq. (46) can be converted to:

$$D = D_{rm}\left[\frac{f}{0,1872}\right]^{1/5} \tag{47}$$

Eq. (47) can be expressed in terms of the correction coefficient "Cc" for the transition from the MRM to the model under study as follows:

$$D = CcD_{rm} \tag{48}$$

Where:

$$Cc = \left[\frac{f}{0,1872}\right]^{1/5} \tag{49}$$

- The relationship between R and R_{rm}:

Using Eqs. (44) and (48), and based on the assumptions mentioned above, the relationship between R *and* R_{rm} can be expressed as follows:

$$R = \frac{4Q}{\pi \nu CcD_{rm}} \tag{50}$$

By substituting Eq. (43) into Eq. (50), the Reynolds number can be rewritten using Eq. (51):

$$R = \left(\frac{4}{Cc\nu}\right)\left(\frac{Q^3 g S_f}{1,4976\pi^3}\right)^{1/5} \tag{51}$$

The combination of Eqs. (3) and (49) leads to Eq. (52):

$$Cc = 1,4\left[-2log\left(\frac{\varepsilon/CcD_{rm}}{3,7}+\frac{5,8}{Cc^{(3/2)}R_{rm}}\right)\right]^{(-2/5)} \quad (52)$$

Eq. (52) is implicit; it can be replaced by Eq. (53), which allows us to obtain an explicit form as follows:

$$Cc_1 = 1,403\left[-2log\left(\frac{\varepsilon}{3,43D_{rm}^{0,9944}}+\frac{5,669}{R_{rm}^{0,9}}\right)\right]^{(-2/5)} \quad (53)$$

The combination of Eqs. (3), (48, 49), and (53), along with simplification, results in Eq. (54), which facilitates the derivation of the explicit equation for the friction factor when the diameter D is the unknown parameter:

$$\frac{1}{\sqrt{f}} = -2log\left[\frac{\varepsilon/D_{rm}}{3,7Cc_1^{0,9666}}+\frac{6}{R_{rm}{Cc_1}^{3/2}}\right] \quad (54)$$

The explicit relationship for the friction factor expressed in (54) was compared with Eq. (3). The maximum relative error recorded, as shown in Fig. 6, was 0,086 % across the entire range of the Moody diagram, making it an interesting result, as illustrated in Fig. 5.

In contrast, for Eq. (55) proposed by Achour and Bedjaoui [5], the maximum relative error achieved was 0,185 %, as reported in Fig. 6.

$$f_r = \left[-2log\left(\frac{\varepsilon/D_r}{3,7\tau}+\frac{10,04}{R_r\tau^{3/2}}\right)\right]^{-2} \quad (55)$$

For the computation of the pipe diameter, Eq. (56) is proposed, which combines Eqs. (2) and (54):

$$D = \left[\frac{8Q^2}{g\pi^2 j}\right]^{1/5}\left[-2log\left[\frac{\varepsilon/D_{rm}}{3,7Cc_1^{0,9666}}+\frac{6}{R_{rm}{Cc_1}^{3/2}}\right]\right]^{-2/5} \quad (56)$$

Using Eqs. (48), (49), and (54), the pipe diameter can be expressed as follows:

$$D = D_{rm}\left[\frac{1}{0.1872}\right]^{1/5}\left[-2log\left[\frac{\varepsilon/D_{rm}}{3,7Cc_1^{0,9666}}+\frac{6}{R_{rm}{Cc_1}^{3/2}}\right]\right]^{-2/5} \quad (57)$$

The Eqs. (56) and (57) are highly accurate, with the maximum relative error recorded being approximately 0,017 %, making them the most precise formulas among the existing ones. These equations are validated for the entire range of the Moody diagram, as shown in Fig. 7.

The maximum relative error for the case of smooth turbulent flow, $\varepsilon/D = \varepsilon/Cc_1D_{rm} = 0$, using Eq. (60), is reduced to 0,0085 % (8, 5.10^{-3}%), as shown in Fig. 8.

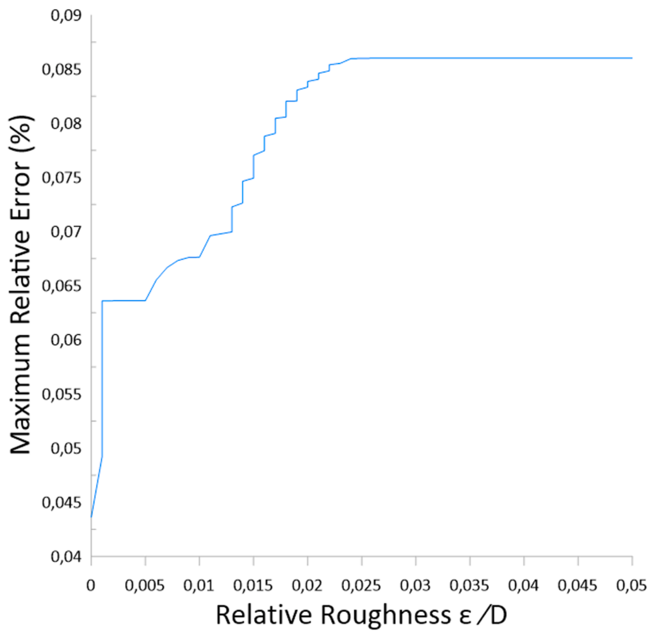

Fig. 5. Maximum relative errors (%) for the Eqs. (54). For the range: $0 \le \varepsilon/D = \varepsilon/(Cc_1D_{rm}) \le 5.10^{-2}$, $R \ge 2300$ and $Q \ge 0,5l\ /sec$.

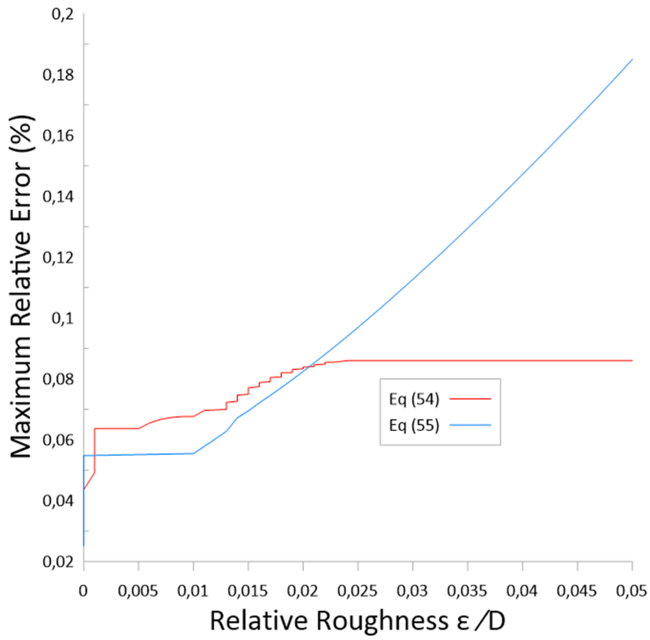

Fig. 6. Maximum relative errors (%) for the Eqs. (54) and (55). For the range: $0 \leq \varepsilon/D \leq 5.10^{-2}$, $R \geq 2300$rem.

$$Cc_2 = 1,403\left[-2log\left(\frac{5,669}{R_{rm}^{0.9}}\right)\right]^{(-2/5)} \tag{58}$$

$$\frac{1}{\sqrt{f}} = -2log\left[\frac{6}{R_{rm}Cc_2{}^{3/2}}\right] \tag{59}$$

$$D = \left[\frac{8Q^2}{g\pi^2 j}\right]^{1/5}\left[-2log\left(\frac{6}{R_{rm}Cc_2{}^{3/2}}\right)\right]^{-2/5} \tag{60}$$

In the case of smooth flow where $\varepsilon = 0$, Eq. (57) can be reformulated as follows:

$$D = D_{rm}\left[\frac{1}{0.1872}\right]^{1/5}\left[-2log\left[\frac{6}{R_{rm}Cc_1{}^{3/2}}\right]\right]^{-2/5} \tag{61}$$

Comparing Eqs. (17) and (34), Eq. (34) is highly accurate, with a maximum relative error of 0,016 %. However, Eq. (60) or (61) recorded an even lower maximum relative error, as shown in Figs. 8 and 9.

Compared to other methods, the proposed formulas consistently achieve minimal error values, with MAE and RMSE at 0,0001 for the entire range and smooth regime, which underscores their accuracy. This precision, coupled with a near-perfect R^2 and Pearson's R of 1, signifies excellent predictive capability. These formulas also show reduced Bias and variability, evident in their low SD and CV, which enhance reliability under diverse conditions.

Moreover, the proposed formulas excel in different flow regimes, adapting to the smooth and transitional zones with minimal relative errors. This adaptability ensures broader applicability compared to methods tailored for specific conditions. The analysis underscores the superior precision and reliability of the proposed formulas, making them a preferred choice for hydraulic calculations across varied scenarios. The minimal error margins and robust statistical performance provide a clear advantage, reducing uncertainty in critical engineering applications.

The following steps summarize how the pipe diameter can be calculated when $Q, S, \varepsilon,$ and ν are known using the MRMM:

- ➢ Using the known parameters, the diameter of the MRM D_{rm} can be determined using Eq. (43).
- ➢ The Reynolds number of the MRM R_{rm} can be estimated based on Eqs. (44).
- ➢ Based on the known value of the absolute roughness ε and the diameter of the MRM D_{rm}, the value of the relative roughness can be deduced.
- ➢ After that, the correction coefficient Cc_1 can be computed using Eq. (53).
- ➢ The friction factor, based on Eq. (54), is directly obtained.
- ➢ Finally, the diameter can be estimated using Eqs. (56) or (57).

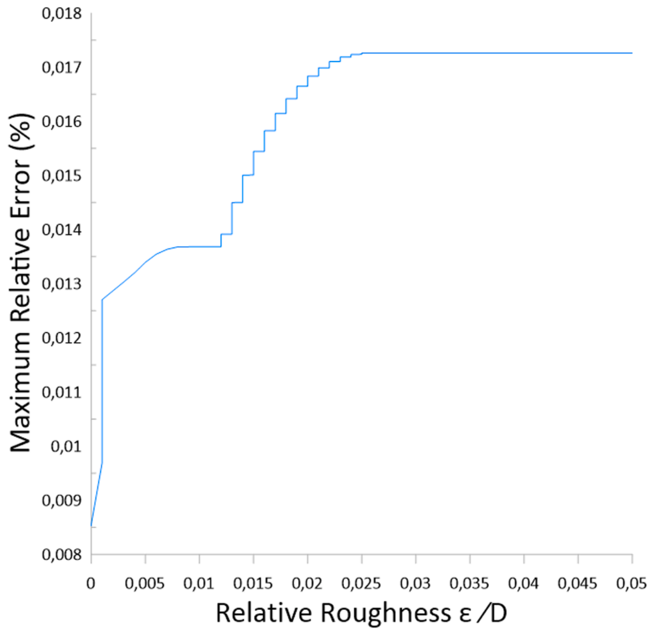

Fig. 7. Maximum relative errors (%) for the Eqs. (55) and (56) For the range: $0 \le \varepsilon/D = \varepsilon/Cc_1D_{rm} \le 5.10^{-2}$, $R \ge 2300$, $Q \ge 0,5l\ /sec$.

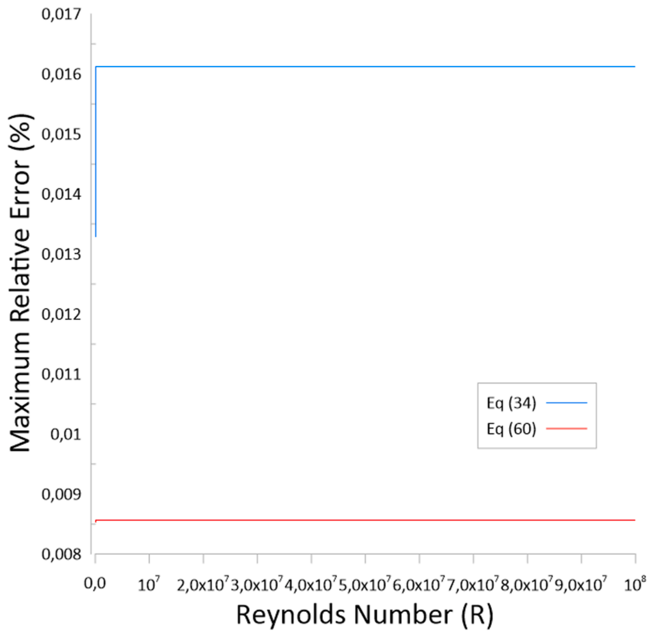

Fig. 8. Maximum relative errors (%) for the Eqs. (34) and (60) or (61) For the range: Eq. (34): $\varepsilon/D = 0$, $3005 \le R \le 10^8$. Eq. (60): $\varepsilon\ /D = \varepsilon\ /Cc_1D_{rm} = 0$, $2300 \le R \le 10^8$.

Conclusion

A comprehensive review led to a novel formula for direct pipe diameter computation using the Modified Rough Model Method (MRMM). Proposed Eqs. (56) and (60) demonstrated superior performance, with maximum relative errors of 0,017 % and 0,0085 %, respectively. An explicit friction factor Eq. (54) was also derived with an accuracy of 0,086 %.

Statistical analysis underscores the formulas' superiority. Eqs. (56) and (60) exhibited minimal errors, with MAE and RMSE values as low as 0,0001, and achieved perfect correlation ($R^2 = 1$). The friction factor formula (54) also showed strong statistical reliability, with low variability (CV) and negligible Bias.

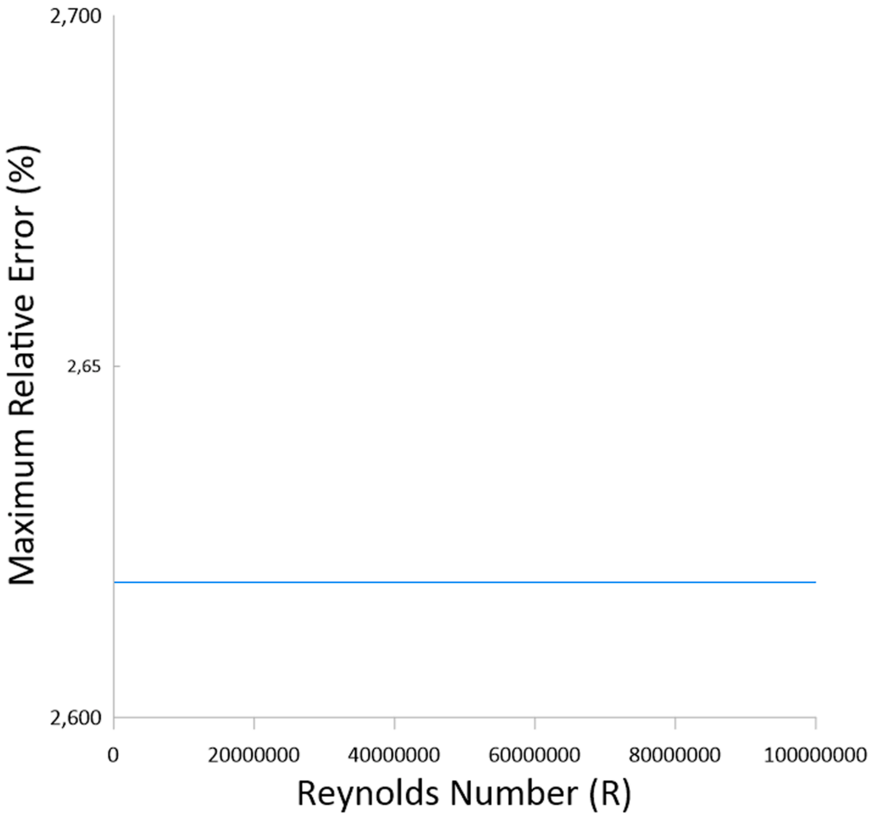

Fig. 9. Maximum relative errors (%) for the Eq. (16). For the range: $\varepsilon/D = 0$, $2450 \leq R \leq 10^8$.

These results yield clear recommendations for practical implementation and future scientific inquiry. The immediate adoption of these explicit, non-iterative formulas is advised for the hydraulic design of piping systems, as they eliminate convergence issues and enhance computational efficiency. Consequently, integration into fluid dynamics software and digital twin platforms is recommended to supersede traditional iterative solvers.

For future work, it is recommended to extend the MRMM methodology to cases where the flow rate or the head loss per unit length is the unknown parameter. Further extension to more complex scenarios like non-Newtonian fluids, compressible flows, and multi-phase flow networks is also advised. Incorporation of these formulas into engineering curricula is recommended to bridge classical theory and modern computational practice. The formulas represent highly reliable and precise tools for pipe flow hydraulics.

CREDIT author statement

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All authors read and approved the final manuscript.

Declaration of competing interest

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Supplementary materials

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