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# Exponential decay and numerical solution of nonlinear Bresse-Timoshenko system with second sound

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## ABSTRACT

This paper aims to study the one-dimensional nonlinear Bresse-Timoshenko system with second sound where the heat conduction given by Cattaneo's law is effective in the second equation. We prove that the system is exponentially stable by using the energy method that requires constructing a suitable Lyapunov functional through exploiting the multipliers method. Furthermore, the result does not depend on any condition on the coefficients of the system. Finally, we validate our theoretical result by performing some numerical approximations based on the standard finite elements method, by using the backward Euler scheme for the temporal and spatial discretization.

## ARTICLE HISTORY

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## KEYWORDS

Energy method; exponential decay; finite elements method; nonlinear Bresse-Timoshenko system; numerical approximation; second sound

## 1. Introduction and position of problem

In the present paper, we consider the following one-dimensional nonlinear Bresse-Timoshenko system with second sound

$$\begin{cases} \rho_1 \varphi_{tt} - k(\varphi_x + \psi)_x + \mu_1 \varphi_t = 0 & \text{in } (0, 1) \times (0, \infty), \\ -\rho_2 \varphi_{txx} - b\psi_{xx} + k(\varphi_x + \psi) + \gamma \theta_x + f(\psi) = 0 & \text{in } (0, 1) \times (0, \infty), \\ \rho_3 \theta_t + kq_x + \gamma \psi_{tx} + \lambda \theta = 0 & \text{in } (0, 1) \times (0, \infty), \\ \tau_0 q_t + \delta q + k\theta_x = 0 & \text{in } (0, 1) \times (0, \infty). \end{cases} \quad (1.1)$$

With the initial and boundary conditions

$$\begin{cases} \varphi(x, 0) = \varphi_0(x), \varphi_t(x, 0) = \varphi_1(x), \psi(x, 0) = \psi_0(x) & \text{in } (0, 1), \\ \psi_t(x, 0) = \psi_1(x), \theta(x, 0) = \theta_0(x), q(x, 0) = q_0(x) & \text{in } (0, 1), \\ \varphi(0, t) = \varphi(1, t) = \psi(0, t) = \psi(1, t) = q(0, t) \\ = q(1, t) = \theta(0, t) = \theta(1, t) = 0 & \text{in } (0, \infty), \end{cases} \quad (1.2)$$

where  $t \in (0, +\infty)$  denotes the time variable and  $x \in (0, 1)$  is the space variable along with the beam of length  $L$ , in its equilibrium configuration. Here  $\varphi$ ,  $\psi$ ,  $\theta$ ,  $q$  and  $f(\psi)$  are specific functions represent, respectively, the transverse displacement of the beam, the rotation angle, the different temperatures, the heat flux and forcing term. The coefficients  $\rho_1$ ,  $\rho_2$ ,  $\rho_3$ ,  $\mu_1$ ,  $\tau_0$ ,  $\delta$ ,  $\gamma$ ,  $b$ ,  $k$  and  $\lambda$  are positive constants that represent the constitutive parameters defining the coupling among the different components of the materials.