



A fast iterative regularization method for ill-posed problems

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Abstract

Ill-posed problems manifest in a wide range of scientific and engineering disciplines. The solutions to these problems exhibit a high degree of sensitivity to data perturbations. Regularization methods strive to alleviate the sensitivity exhibited by these solutions. This paper presents a fast iterative scheme for addressing linear ill-posed problems, similar to nonstationary iterated Tikhonov regularization. Both the a-priori and a-posteriori choice rules for regularization parameters are provided, and both rules yield error estimates that are order optimal. In comparison to the nonstationary iterated Tikhonov method, the newly introduced method significantly reduces the required number of iterations to achieve convergence based on an appropriate stopping criterion. The numerical computations provide compelling evidence regarding the efficacy of our new iterative regularization method. Furthermore, the versatility of this method extends to image restorations.

Keywords Ill-posed problems · Iterative regularization · Error estimates · Image deblurring

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1 Introduction

We investigate inverse problems characterized by the equation

$$Ax = y^\delta, \quad (1.1)$$

where A represents a bounded linear operator from a Hilbert space X to Hilbert space Y , possessing a non-closed range. The variable y^δ represents the measured data, which

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