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An Implicit Iteration Method for Solving Linear Ill-Posed Operator Equations

T. Bechouat*

Department of Mathematics and Informatics, Mohammed Cherif Messaadia University, Faculty of Science and Technology, Souk Ahras, Algeria

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Abstract—In this work, we study a new implicit method to computing the solutions of ill-posed linear operator equations of the first kind under the setting of compact operators. The regularization theory can be used to demonstrate the stability and convergence of this scheme. Furthermore, we obtain convergence results and effective stopping criteria according to Morozov's discrepancy principle. The numerical performances are conducted to show the validity of our implicit method and demonstrate its applicability to deblurring problems.

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1. INTRODUCTION

We investigate the solution of the following operator equations of the first kind

$$Ax = y, \tag{1}$$

where $A : X \to Y$ is a compact operator with nonclosed range ($\overline{R(A)} \neq R(A)$) and X, Y are separable Hilbert spaces.

Problems (1) arise in several areas of science and engineering for example if A is an integral operator, the equations of this form arise in many applications, including remote sensing, computerized tomography and deblurring images, to name a few.

We denote by (σ_i, u_i, v_i) the singular system of A, i.e., (σ_i) is a sequence of positive real numbers such that $\sigma_i \to 0$ and $\{u_i\}$, $\{v_i\}$ are orthonormal basis of orthogonal complement to the null space $N(A)$ and $\overline{R(A)}$ respectively.

Let $x \in X$ and $y \in Y$, it is clear that

$$Ax = \sum_{i=1}^{\infty} \sigma_i \langle x, u_i \rangle v_i, \tag{2}$$

and

$$A^* y = \sum_{i=1}^{\infty} \sigma_i \langle y, v_i \rangle u_i, \tag{3}$$

where $A^* : Y \to X$ is the adjoint of the operator A.

*E-mail: t.bechouat@gmail.com